satellite applications to the study of the atmosphere and oceans is contained in Baker (1990).

Not all satellites orbit in polar or geostationary orbits. Since much of the former Soviet Union lies in high latitudes, the use of geostationary satellites orbiting in the equatorial plane is useless for their communication needs. Soviet communication satellites were placed in a highly elliptical orbit (called the Molynia orbit, Kidder and Vonder Haar, 1993).

Satellites do not remain in their orbits indefinitely. Ultimately, loss of energy through the action of atmospheric drag causes the satellite to loose speed and fall back to Earth, or burn up in the atmosphere. This action therefore provides an upper limit to the useful lifetime of the satellite. We can estimate this lifetime by considering the reduction on the radius of the satellite $\delta r$ in a circular orbit

$$\delta r \approx 4\pi d A \rho_{air} R^2/m$$

where $d$ is a drag coefficient, $A$ is the satellite's cross–sectional area (normal to its motion), and $\rho_{air}$ is the atmospheric density at satellite altitude. The uncertainty in this expression hinges on the uncertainty to which the drag coefficient is known and this is probably only accurate to within a factor of 2. For example, consider the Landsat 5 satellite. The mass of this satellite is about 1700 kg, its area $A$ is about 10 $m^2$ and its orbital altitude is 700 km where the atmospheric density is about $10^{-13}$ kg $m^{-3}$. Thus the satellite descends about 0.4 m per orbit, or about 5 m per day and we infer that the maximum useful time of Landsat 5 in orbit (not the operation of instruments on Landsat) is in excess of 100 years. By contrast, the low orbit of the TRMM satellite limits its usefulness to only a few years.

The general concept of the Tropical Rainfall Measurement Mission (TRMM), the satellite orbit and its implications for sampling the tropical atmosphere is given in Simpson et al. (1988).

# 2
# The Nature of Electromagnetic Radiation

For the topics considered in this book, it is electromagnetic radiation, in the form of a wave, that communicates information from the atmosphere to the observer. The light that we detect by our eye is such a wave. This chapter deals with the fundamental properties of such waves: the frequency at which they oscillate, the way in which they propagate, and the manner by which they are created. These properties form an important basis for understanding the way that electromagnetic waves interact with matter, and, therefore, the way we utilize this radiation for remote sensing.

Electromagnetic waves are generated by oscillating (or, more generally, time varying) electric charges which, in turn, generate an oscillating electric field. A characteristic of an oscillating electric field is that it produces an accompanying oscillating magnetic field that further produces an oscillating electric field. Therefore these fields, initiated by the oscillating charge, proceed outward from the original charge, each creating the other. A visualization of such a propagating wave is given in Fig. 2.1. It was James Clerk Maxwell, who, more than a century ago, provided us with the theoretical synthesis of this phenomenon. While detailed mathematical accounts of this work are omitted here, they can be found in most standard texts on electromagnetics.

## *Excursus: The Electric Dipole*

Oscillating charges produce electromagnetic radiation. A particular charge distribution of basic importance to our understanding of radiation and its interaction with matter is the *electric dipole*. The electric dipole consists of two charges: a positive and a negative charge of the same magnitude $q$, separated by a distance $s$ (Fig. 2.2a). From this concept emerges the definition of the *dipole moment*

$$\vec{p} = q\vec{s} \tag{2.1}$$

which is a vector of magnitude $qs$ directed from the negative to the positive charge. Oscillation of this dipole produces an electromagnetic wave which has properties that are discussed more fully below.

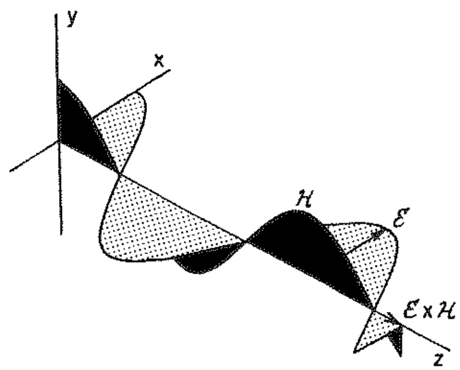


**Figure 2.1** A schematic view of a time harmonic electromagnetic wave propagating along the $z$ axis. The oscillating electric $\mathcal{E}$ and magnetic $\mathcal{H}$ fields are shown. Note that the oscillations are in the x–y plane and perpendicular to the direction of propagation.

The concept of an oscillating electric charge is fundamental to our understanding of particle scattering. There are, however, other arrangements of charges that also oscillate and thus contribute to the electromagnetic field emerging from matter. These charge distributions can be considered as *multipoles* and are defined as follows. A single point charge is a *monopole.* A dipole is obtained by displacing a monople through a small distance ($s$ in Fig. 2.2a) and replacing the original monopole by another, but of the opposite sign. Likewise, a *quadrupole* is obtained by displacing a dipole a small distance and replacing this dipole with one of opposite sign (Fig. 2.2b). This idea can be continued to build up higher order multipoles. In this way, the charge distribution in a particle can be considered as a composition of various orders of multipoles. We shall see later how the oscillation of these multipoles contribute to the radiation scattered by a particle.

## 2.1 The Electromagnetic Spectrum

Electromagnetic theory predicts that the electromagnetic wave travels at a unique speed $c$, which is the speed of light. The wavelength of the wave depends upon how rapidly, or the *frequency* at which, the charge oscillates. We shall denote this frequency (number of



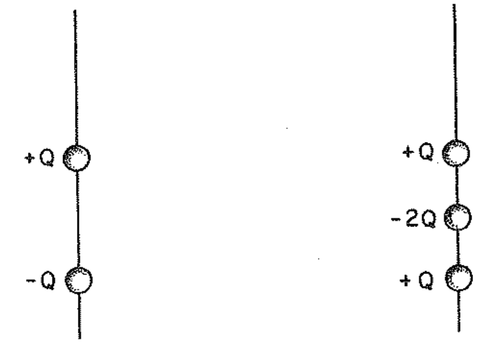


**Figure 2.2** (a) The two charges $\pm q$ form a dipole. The dipole moment is a vector which has a magnitude of $qs$ directed toward the positive charge. (b) Charges $+q$ and $-2q$ are arranged along a line to form an axial quadrupole.

oscillations per second) by $\nu$, and it is related to $c$ by

$$\nu = \frac{c}{\lambda} \tag{2.2}$$

where $\lambda$ is the wavelength of the wave. For example, red light with a wavelength of 0.7 micrometers ($\mu$m) corresponds to a frequency of $4.3 \times 10^{14}$ oscillations per second while violet light, at 0.4 $\mu$m, corresponds to $7.5 \times 10^{14}$ oscillations per second. An alternate way of describing the frequency of radiation is in terms of *wavenumber*

$$\tilde{\nu} = \frac{1}{\lambda} \tag{2.3}$$

which is a count of the number of wave crests or troughs in a given unit of length. For example, red light has 14,286 wave crests in a centimeter whereas 25,000 crests can be counted in a centimeter of violet light. Wavenumber is the measure often used by spectroscopists and others involved in experimental measurements of the interaction of radiation with matter.

Even before Maxwell, the spectrum of electromagnetic radiation (that is the range of wavelengths or frequencies of the radiation) was extended beyond the visible (i.e., beyond those wavelengths detectable by the human eye). In fact, we now know that the visible portion of the spectrum, from 0.4 to 0.7 $\mu$m, is just a tiny part of a much broader spectrum of electromagnetic radiation. The various portions of the spectrum and the terminology commonly used to refer to these regions are given in Fig. 2.3.

The remote sensing techniques addressed in this book are generally concerned with the portion of the spectrum from the ultraviolet to the microwave and radiowavelength regions. A sense of the importance of the various spectral regions to present methods of atmospheric remote sensing is also provided in Fig. 2.3, which shows the considerable range and variety of sensors flown on the Nimbus 7 experimental satellite.

## 2.2 Wave Propagation

### 2.2.1 Mathematical Description

Figure 2.1 provides a snapshot of an electromagnetic wave and illustrates how the electric and magnetic fields move back and forth as the wave travels along its path. An obvious characteristic of this propagation is that these fields repeat themselves at set distances along the path; this distance is the wavelength of the radiation. In the most general sense, suppose we have a displacement $\mathcal{E}$ (for our purposes, this is the magnitude of the electric field) which is specified as a function $\mathcal{E} = f(x)$. The displacements at the point $x \pm \lambda$ are the same as at $x$. We conclude that a mathematical expression of the form

$$\mathcal{E} = f(x \pm ct) \tag{2.4}$$

adequately describes the physical situation that repeats as it travels or propagates along the $\pm x$ directions. While $\mathcal{E}$ is taken here to represent the electric field, $\mathcal{E}$ may in fact represent a great diversity of physical quantities, such as a deformation in the Earth's crust, the pressure of a gas, and many others.

An especially important example of repetitive wavelike motion is the harmonic wave which is described by the formula

$$\mathcal{E} = \mathcal{E}_o \cos k(x - ct) \tag{2.5a}$$

The quantity $\mathcal{E}_o$ has a special meaning. It is the amplitude of the wave and, as we shall see later, the energy carried by the wave is

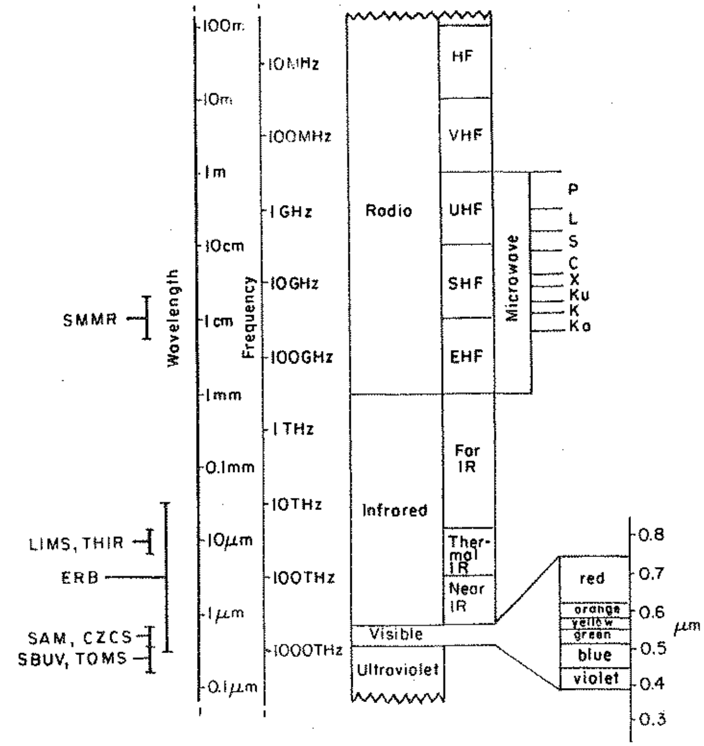


**Figure 2.3** The electromagnetic spectrum. The diagram shows those parts of the electromagnetic spectrum which are important in remote sensing, together with the conventional names of the various regions of the spectrum. The letters (P, L, S, etc.) used to denote parts of the microwave spectrum are in common use in remote sensing, being standard nomenclature among radar engineers in the United States. Various terminologies are in use for the subdivisions of the infrared (IR) part of the spectrum. That adopted here defines the thermal band as lying between 3 and 15 $\mu$m, since this region contains most of the power emitted by black bodies at terrestrial temperatures. Also shown are wavelength regions of sensors on the Nimbus 7 satellite.

related to the square of this amplitude. The quantity $k(= 2\pi\tilde{\nu})$ is also referred to as wavenumber, but this should not prove to be a source of confusion as $\tilde{\nu}$ and $k$ are used in different contexts; $k$ generally applies to wave propagation, whereas $\tilde{\nu}$ is used, as in the previous section, to discriminate regions of the electromagnetic spectrum. Equation (2.5a) can also be written in the form

$$\mathcal{E} = \mathcal{E}_o \cos(kx - \omega t) \qquad (2.5b)$$

where $\omega = kc = 2\pi c/\lambda$ is the angular frequency of the wave and, according to (2.2), $\omega = 2\pi\nu$.

The argument of the cosine function in (2.5a) also has a particular meaning. It is represented by the function $\phi$

$$\phi = k(x - ct) \qquad (2.6)$$

and is referred to as the *phase* of the wave. If, for instance, the plane wave example of Fig. 2.4 is considered, then the plane that contains the wave crests is also a plane over which the phase is the same at all points. This situation is most easily visualized by the analogy to a plane surface wave such as the one shown in Fig. 2.4.

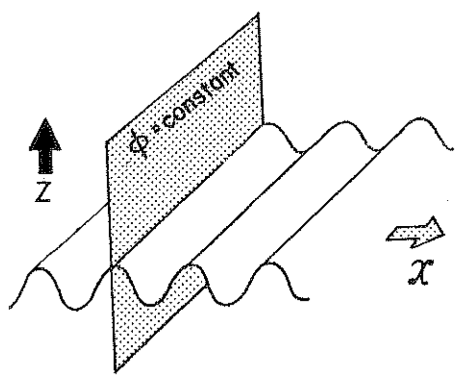


**Figure 2.4** An undulating surface as an analogy to a propagating plane wave. All points along lines of equal displacement (such as along the ridges of the surface) correspond to lines of equal phase and are referred to as wavefronts. A light "ray" is simply the line drawn normal to these fronts. This ray can be characterized by the unit vector $\vec{n}$ along the direction of the ray.



The mathematical form of the harmonic wave can be written in a more general way by introducing complex variables and by noting, through the use of Maclaurin's theorem, that

$$e^{\pm i\phi} = \cos\phi \pm i \sin\phi$$

from which it follows that (2.5a) becomes

$$\mathcal{E}(x,t) = \mathcal{E}_o e^{ik(x-ct)} \qquad (2.7)$$

and it is taken for granted that the real part of this expression represents the wave. The general representation of the harmonic wave requires that the displacement $\mathcal{E}$ be specified at $x = 0$ and $t = 0$. We specify this initial displacement (initial since it is defined at $t = 0$) in terms of a constant phase $\phi_o$ at $x = 0$ and $t = 0$. For this general case,

$$\phi = \phi_o + k(x - ct) \qquad (2.8)$$

and

$$\mathcal{E}(x,t) = \mathcal{E}_o e^{i\phi} \qquad (2.9)$$

Simple algebraic manipulations show that the square of the wave amplitude, given as

$$|\mathcal{E}(x,t)|^2 = |\mathcal{E}_o|^2 \qquad (2.10)$$

is the same for all $x$ and $t$ since $\mathcal{E}_o$ is a constant. The energy transferred by the wave, related to $|\mathcal{E}_o|^2$, does not vary along its path of propagation and is independent of our definition of $\phi_o$. It is only the interaction of the wave with matter that alters the energy of a propagating wave. It is ultimately this energy modulation that is exploited in remote sensing.

## *Excursus: The Intensity and Irradiance of Electromagnetic Radiation*

An electromagnetic wave, traveling through space at the speed of light, carries electromagnetic energy which is detected by sensors that respond to this energy. Energy flows in the direction in which the wave advances and this direction of propagation is defined by the vector cross-product $\vec{\mathcal{E}} \times \vec{\mathcal{H}}$. The energy per unit area per unit time flowing perpendicular into a surface in free space is given by the *Poynting vector* $\vec{\mathcal{S}}$, where

$$\vec{\mathcal{S}} = c^2 \epsilon_o \vec{\mathcal{E}} \times \vec{\mathcal{H}}$$

where $c$ is the speed of light and $\epsilon_o$ is the vacuum permittivity. Energy per unit time is power, so the SI units of $\vec{\mathcal{S}}$ are $\mathrm{Wm}^{-2}$. At the frequencies of interest to the topics of this book, the fields $\vec{\mathcal{E}}, \vec{\mathcal{H}}$, and $\vec{\mathcal{S}}$ oscillate at rapid rates and it remains impractical to measure an instantaneous value of $\vec{\mathcal{S}}$ directly. We measure its average magnitude $< \mathcal{S} >$ over some time interval that is a characteristic of the detector. This time averaged quantity is referred to as the *radiant flux density*.

Strictly speaking, the flux density emerging from the surface is known as the *exitance* and the flux density incident on the surface is called the *irradiance*. To avoid unnecessary complications with nomenclature, we refer to the flux density onto or from a surface as either irradiance or flux and use the symbol $F$ to represent this quantity.

When the flow of light is nonparallel and when the detector collects the light confined to a range of directions, specified by a small element of solid angle $d\Omega$, then the quantity sensed is the *intensity*, defined as $< \mathcal{S} > /d\Omega$ and has units of $\mathrm{Wm}^{-2}\mathrm{ster}^{-1}$. This is a quantity that is used throughout this book and we will denote it by the symbol $I$.

We can consider a more direct relationship between the energy carried by an electromagnetic wave and the amplitudes of the electric and magnetic fields by considering the simple case of a plane wave of the form

$$\vec{\mathcal{E}} = \vec{\mathcal{E}}_o \cos(kx - \omega t)$$

The magnetic field also has the form $\vec{\mathcal{H}} = \vec{\mathcal{H}}_o \cos(kx - \omega t)$ and therefore

$$\mathcal{S} = c^2 \epsilon_o \vec{\mathcal{E}} \times \vec{\mathcal{H}} = c^2 \epsilon_o \vec{\mathcal{E}}_o \times \vec{\mathcal{H}}_o \cos^2(kx - \omega t)$$

Hence

$$< \mathcal{S} > = c^2 \epsilon_o \mid \vec{\mathcal{E}}_o \times \vec{\mathcal{H}}_o \mid < \cos^2(kx - \omega t) >$$

and the time average is calculated for an interval of length $T$ according to

$$< \cos^2(kx - \omega t) > = \frac{1}{T} \int_t^{t+T} \cos^2(kx - \omega t')dt'$$

$$= \frac{1}{2} - \frac{1}{4\omega T}[\sin(2kx - 2\omega(t+T)) - \sin 2(kx - \omega t)]$$

When $T \gg t$, $\omega T \gg 1$ and $< \cos^2(kx - \omega t) > \rightarrow 1/2$. Since $\mathcal{E}_o = c\mathcal{H}_o$,

$$F = < \mathcal{S} > \approx \frac{c\epsilon_o}{2} \mathcal{E}_o^2$$



or

$$F \approx c\epsilon_o < \mathcal{E}^2 >$$

where $< \mathcal{E}^2 > = \mathcal{E}_o/2$.

### 2.2.2 Waves in Three Dimensions

Although our simple relation $\mathcal{E} = f(x - ct)$ represents a wave motion propagating along the $x$ axis, it does not mean that a wave is actually concentrated *on* the axis. If we consider the physical disturbance extended over all space at a specified time $t$, then the function $\mathcal{E} = f(x - ct)$ takes the same value at all points having the same $x$. In three-dimensional space, $x =$ *constant* represents a plane perpendicular to the $x$ axis such as demonstrated in Fig. 2.5a. Thus $\mathcal{E} = f(x - ct)$ represents a propagating plane wave in three dimensions.

Suppose that instead of propagating along the $x$ axis, this plane wave propagates along a general direction characterized by the unit vector $\vec{n}$. If $\vec{r}$ is the position vector of a point on the wave front, then $\vec{n} \cdot \vec{r}$ is the distance measured from an origin along the direction of propagation. Thus we write

$$\mathcal{E} = f(\vec{n} \cdot \vec{r} - ct) \tag{2.11}$$

for the general wave equation. It is convenient to introduce the vector $\vec{k}$ for $k\vec{n}$ in which case the phase of a harmonic wave is

$$\phi = \phi_o + \vec{k} \cdot \vec{r} - \omega t \tag{2.12}$$

A particular type of wave propagation that is important for later considerations is the spherical wave shown in Fig. 2.5c. We can think of this wave as a surface propagating out from a point source like that of a single oscillating dipole. As the wave propagates outward, the wave surface becomes progressively larger (increasing as $r^2$). Since the energy associated with the flow of photons is the same through each wave front, the total energy flowing out from the point source through each spherical surface is proportional to $\mathcal{E}_o^2$. The wave displacement at a point far from the source is approximated by

$$\mathcal{E} = \frac{\mathcal{E}_o}{kr} e^{i\phi}$$

where $\phi$ is again defined according to (2.8). Thus we can conclude that *for a spherical wave, the energy flow per unit area confined to*

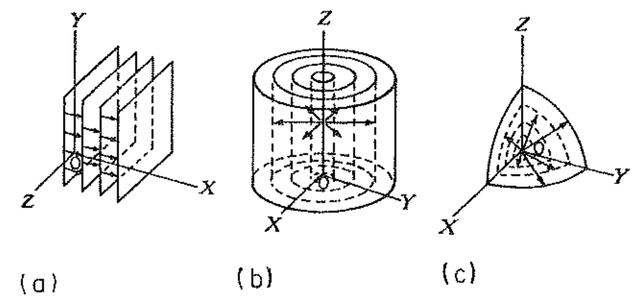


**Figure 2.5** Examples of three-dimensional waves: (a) a plane wave, (b) a cylindrical wave, and (c) a spherical wave.

*a particular direction) decreases as the inverse of the square of the distance from the source.*

### 2.2.3 Doppler Effects

When the source of an electromagnetic wave and an observer are in relative motion with respect to the material medium in which the waves propagate, the frequency of the waves observed is different from the frequency of the emitting source. This is the so-called Doppler effect named after the German-born, Austrian physicist C. J. Doppler, who first noticed the effect in sound waves. The Doppler effect for elastic waves consists of matter in motion and is well described in standard texts on physics. However, the Doppler effect for electromagnetic waves, as exploited in the remote measurement of atmospheric motion using techniques discussed in Chapter 8, requires a treatment different from the more mechanical discussion of this effect. The reasons for this are that electromagnetic waves do not involve matter in motion, and therefore the velocity of the moving source relative to the medium does not enter into discussion. Second, the velocity of propagation is $c$, the speed of light, which is the same for all observers regardless of their relative motion.

Consider the example of one observer at $O'$ moving relative to a source at $O$ with velocity $v$ along the line joining $O$ and $O'$. The Doppler effect for electromagnetic waves is derived from the principle



of relativity which requires that the phase of the wave, $kx - \omega t$, remains invariant when passing from one inertial system (at rest) to another (in motion with a velocity $v$). Therefore,

$$kx - \omega t = k'x' - \omega' t' \tag{2.12}$$

and with arguments that use Lorentz transformations, the frequency observed at $O'$ is shifted relative to the frequency $\nu$ at $O$ according to

$$\nu' = \nu \frac{1 + v/c}{\sqrt{1 - v^2/c^2}} \tag{2.13}$$

For velocities typical of atmospheric motion, $v^2/c^2 << 1$, and

$$\nu' \approx \nu(1 + v/c) \tag{2.14}$$

It is customary to express the Doppler effect in terms of the frequency shift defined relative to the fixed observer;

$$\Delta\nu_D = (\nu' - \nu) = v/\lambda \tag{2.15}$$

where $\Delta\nu_D$ is referred to as the *Doppler shift*. This shift is defined such that $\Delta\nu_D > 0$ when $O'$ is moving toward the observer and negative as $O'$ recedes from the target as in the example of the red shift of white star light of receding galaxies.

When the relative motion is not along the line joining $O$ and $O'$, it follows that

$$\nu' \approx \nu(1 + v\cos\theta/c) \tag{2.16}$$

and

$$\Delta\nu_D = v\cos\theta/\lambda \tag{2.17}$$

where $\theta$ is the angle between the direction of motion and the line connecting the source and observer.

Our applications deal with the frequency shift of an electromagnetic wave scattered by a moving target with the source and the observer stationary relative to each other. To treat this rigorously, the relativistic arguments introduced to derive (2.13) are needed, but the same answer may be obtained using a non-relativistic treatment given a consistent but nonphysical assumption about the motion of the "medium" transmitting the wave motion. Consider the geometry illustrated in Fig. 2.6. Light of frequency $\nu$ from a source at $O$ is scattered by an object at $P$ and observed at $O'$. The angles that the direction of motion make with $OP$ and $PO'$ are $\theta_1$ and $\theta_2$, respectively. The frequency observed at $P$ according to (2.16) is

$$\nu' = \nu(1 + v\cos\theta_1/c) \tag{2.18}$$

If we now consider a hypothetical source at the particle $P$ emitting at $\nu'$ to an observer at $O'$ moving relative to $P$ with a velocity $v\cos\theta_2$, then the frequency of the light received at $O'$ is

$$\nu'' = \nu'(1 + v\cos\theta_2/c) \tag{2.19}$$

Combining (2.18) and (2.19) produces

$$\Delta\nu_D = \nu'' - \nu = \frac{\nu v}{c}(\cos\theta_1 + \cos\theta_2) \tag{2.20}$$

for velocities much smaller than $c$. For a system with a collocated source and receiver (one we refer to as a *monostatic* system), $\theta_1 = \theta_2 = \theta$, and it follows that

$$\Delta\nu = 2v\cos\theta/\lambda \tag{2.21}$$

where $v\cos\theta$ in this sense is the radial velocity of the target relative to the observer. Equation (2.21) provides the basis for wind measurements using the Doppler shift associated with the motion of certain targets (such as aerosol) as they are advected by the wind.

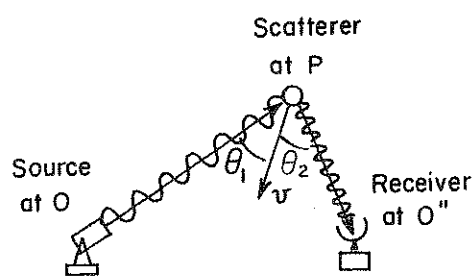


**Figure 2.6** Diagram for the calculation of the Doppler shift on scattering by a moving object at $P$ for a bistatic system.



## 2.3 Polarization

A subtle feature of electromagnetic radiation, and one discovered some two centuries before Maxwell by the peculiar way certain materials reflect light, is the property of *polarization*. This feature played an important role in shaping the electromagnetic theory as it developed during the nineteenth century. Although polarization is not an obvious property of an electromagnetic wave — for example, human vision is not very sensitive to polarized light — it is nevertheless an important property, especially for remote sensing. Two electromagnetic waves, identical in all respects except for their polarization state, can interact differently with matter, and it is the very nature of these differences that are exploited in certain remote sensing methods.

The amount of light transmitted through two sheets of a material arranged as in Fig. 2.7 depends on the rotation of one sheet with respect to the other. For one case, light is transmitted straight through the two sheets although its character is altered on transmission. As one of the sheets is rotated with respect to the other, the intensity of the light decreases until an orientation is reached when no light is transmitted. This type of experiment suggests that an electromagnetic wave has properties in directions other than along the line of propagation and that the wave in some sense is three dimensional. Polarization is a property of this dimensionality.

The previous experiment illustrates the property of polarization. We note in reference to Fig. 2.1 that as the electromagnetic wave propagates along the $z$ axis, the direction of the oscillations of the fields may vary about the direction of propagation. At one point in space the electric vector might be directed along the $y$ axis, while at another point along the beam this vector might have turned to point along the $x$ axis. For the example shown in Fig. 2.7, the polarizing crystal allows only oscillations which are preferentially aligned along a specific direction (the vertical in this example), and the light is said to be linearly polarized along the vertical direction. Although the property of polarization seems to be a subtle property of electromagnetic radiation, it is a property that demonstrates a basic characteristic of electromagnetic waves. It is one that emphasizes the difference between these waves and other types of waves. Electromagnetic waves are *transverse waves* for which the oscillations are perpendicular to the path of propagation.

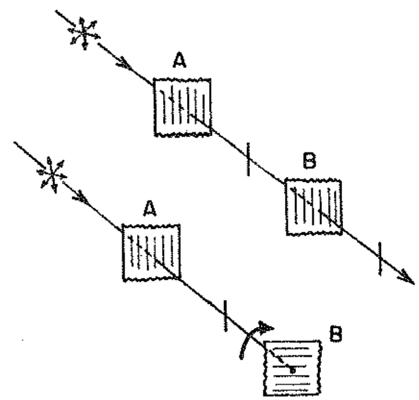


**Figure 2.7** A polarizing crystal A allows only that part of an incident beam that is polarized along a particular direction defined by the properties of the crystal. Polarized light enters the second crystal B which passes the light depending on the relative alignment of A with respect to B. If B is rotated 90 degrees, then none of the polarized beam is transmitted. When used in this way, B is referred to as an analyzer.

### 2.3.1 Mathematical Description

We are now left to devise some way of describing the state of polarized radiation. It is customary to use the behavior of the electric field for this description since the magnetic field is perpendicular to it and a description of one defines the other. The electric field may oscillate in many different ways; it could point in a single direction after transmission through the first of the polarizing crystals as in our proceeding experiment (an example of linear polarization), or it may oscillate in such a way that the superpositions of all directions of oscillations trace a circular pattern, in which case the radiation is said to be *circularly polarized.* There are many more possibilities, but with all of these we can consider that the electric field at any point is simply a superposition of two waves linearly polarized at right angles to each other. Such a decomposition is more than just a mathematical device since, according to our experiment described in relation to Fig. 2.7, polarizers can be used to isolate these perpendicular components and therefore provide a way of analyzing the state of polarized radiation.



We choose to write the electric fields as a superposition of two linearly polarized waves, namely

$$\vec{\mathcal{E}} = \mathcal{E}_\ell \hat{\ell} + \mathcal{E}_r \hat{r} \tag{2.22}$$

where $\mathcal{E}_\ell$ and $\mathcal{E}_r$ are the components of the vibrations along two orthogonal directions shown in Fig. 2.8. The direction vectors $\hat{\ell}$ and $\hat{r}$ are defined so that both are perpendicular to the direction of propagation with the $\hat{\ell}$ component lying parallel to the *plane of reference* and $\hat{r}$ perpendicular to this plane in the sense that $\hat{r} \times \hat{\ell}$ is along the direction of propagation. We will refer to this representation of the $\mathcal{E}$ field as the "linear basis". We could choose other bases to represent the electric field, and an example is discussed later.

When dealing with problems of scattering, it is customary to specify the plane of reference so that it includes both the direction of the incident and the direction of the scattered waves.

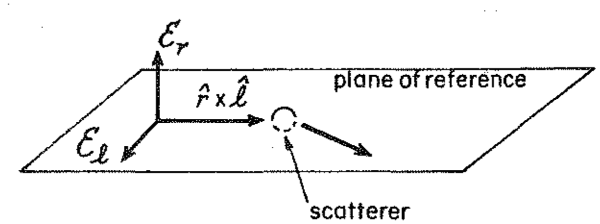


**Figure 2.8** A schematic illustration of the relation between $\hat{\ell}, \hat{r}$, and the direction of propagation of the electromagnetic wave and the plane of reference. The electric field is considered to be a superposition of $\mathcal{E}$ fields along the $\hat{\ell}, \hat{r}$ directions. The symbols $\ell$ and $r$ are taken from the last letters of the words parallel and perpendicular, respectively.

According to (2.5b) we can express each of the orthogonal components in the form

$$\mathcal{E}_\ell = \mathcal{E}_{o,\ell} e^{i(\phi+\phi_\ell)}$$

$$\mathcal{E}_r = \mathcal{E}_{o,r} e^{i(\phi+\phi_r)} \qquad (2.23)$$

where $\phi = kx - \omega t$ and $\phi_r$ and $\phi_\ell$ are the constant phase shifts of the perpendicular and parallel vibrations relative to some origin point. The precise specification of this origin is not important to the mathematical description of polarization as only the difference of the phases of the two orthogonal waves matters. If $\phi_r = \phi_\ell$, then (2.23) defines a straight line and the radiation is said to be *linearly polarized*. We can mathematically represent this linearly polarized radiation by eliminating the harmonic factor in (2.23) to give

$$\mathcal{E}_r = (\mathcal{E}_{o,r}/\mathcal{E}_{o,\ell})\mathcal{E}_\ell \qquad (2.24)$$

where the slope of the line is $\mathcal{E}_{o,\ell}/\mathcal{E}_{o,r}$. Figures 2.9a and b illustrate examples of the electric field vector projected on the plane normal to the direction of propagation for two cases of linear polarization. For the more general case when the phase constants differ by a fixed amount, then the polarization is no longer linear. Examples of this situation are presented in Figs. 2.9c to f for phase differences of either 90 or 45 degrees. When both the phases and the amplitudes of the fields differ in a fixed way, the electromagnetic wave is said to be *elliptically polarized* as in the cases shown by Figs. 2.9d to f.

In summary, we are able to describe the state of polarization of an electromagnetic wave in terms of two linearly polarized waves vibrating at right angles to each other with some fixed phase difference. We both visualize and describe this state of polarization as a projection on two perpendicular axes of the electric vector rotating with a circular frequency $\omega$ around the direction of propagation. Figure 2.10 provides a clear visualization of this idea for the case of two equal amplitude waves out of phase by 90 degrees from each other. The position of the electric vector as the wave propagates along the $z$ axis is also drawn at eight different locations along the path. In this case, the tip of the vector rotates to the right as the beam propagates along. As seen by an observer looking toward the light, the electric field rotates counterclockwise.

In defining the complete state of polarization only three pieces of information are required. These are the amplitudes of each wave and the phase difference between them. A fourth parameter, the



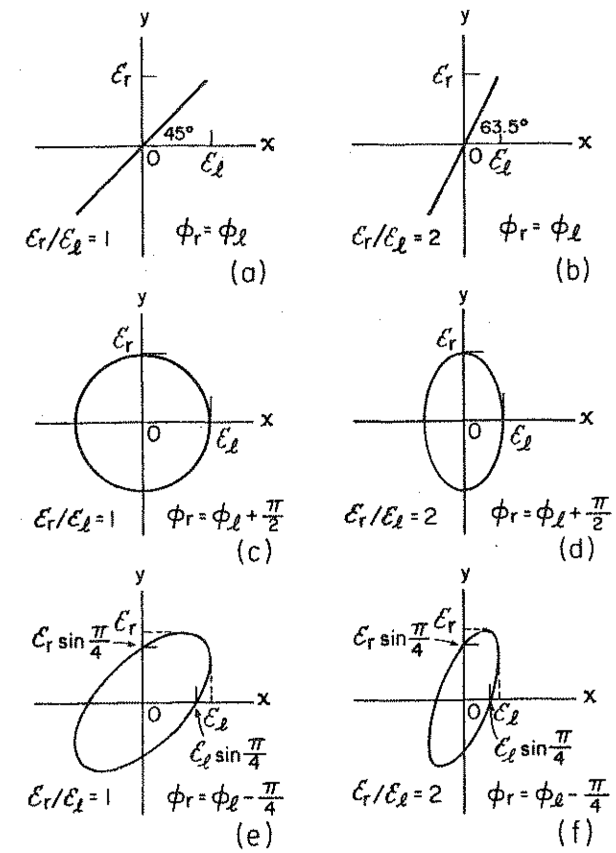


**Figure 2.9** Simple harmonic motions in two dimensions: (a) The amplitudes along the $r$ and $\ell$ directions are the same, as are their phase constants. (b) $r$s amplitude is twice $\ell$s but their phase constants are the same. (c) Their amplitudes are equal, but $\ell$ leads $r$ in phase by 90 degrees. (d) Same as (c), but $r$'s amplitude is twice $\ell$s. (e) Equal amplitudes, but $\ell$ lags $r$ in phase by 45 degrees. (f) Same as (e) except $r$s amplitude is twice $\ell$s.

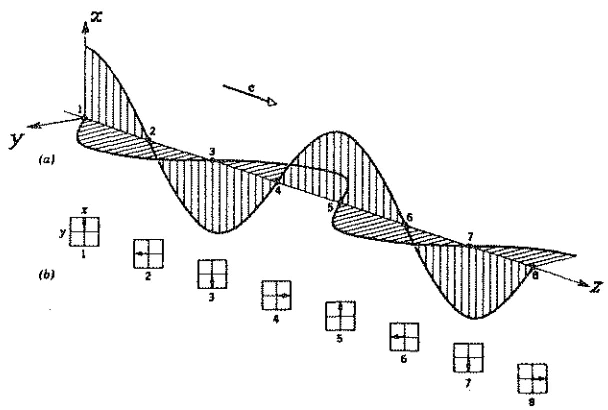


**Figure 2.10** (a) Two linearly polarized waves of equal amplitude at right angles to each other moving along the $z$ axis. For the case drawn, they differ in phase by 90 degrees; where one wave has a maximum value, the other is zero. (b) Views of the resultant amplitude of the approaching wave as seen by observers located at the positions shown on the $z$ axis. The electric vector moves to the right as seen lookin along the $z$ axis. The electromagnetic wave is said to be right-handed circularly polarized.

sign of the phase difference, defines the sense by which the vector rotates and is referred to as the *handedness* of the polarization. The polarization is said to be right-handed if the $\mathcal{E}$ field at an instant in time traces out a right-hand screw or spiral and left-handed if the $\mathcal{E}$ traces out a left-handed screw.

### 2.3.2 Examples of Polarized Radiation

Electromagnetic waves in the radio and microwave range are generated by surging a charge up and down a wire to create an oscillating dipole in the form of a transmitting antenna as shown in Fig 2.11. The field transmitted by such a dipole tends to be linearly polarized in a direction parallel to the dipole axis. When this polarized wave in turn falls on a receiving antenna, the alternating electric field of the transmitted wave causes the charges in the receiving antenna to



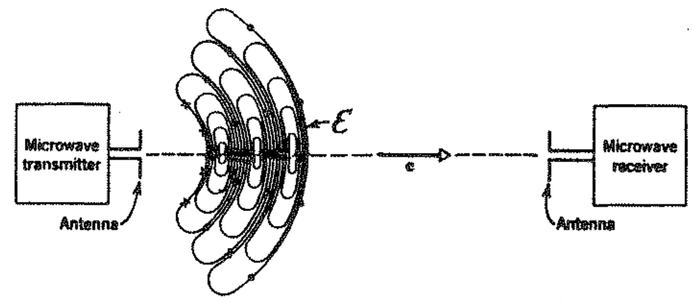


**Figure 2.11** The $\mathcal{E}$ vectors of the linearly polarized transmitted electromagnetic wave are parallel to the axis of the receiving antenna so that a wave will be detected. If the antenna is rotated through 90 degrees about the direction of propagation, no signal is detected. The electric field radiates out from the transmitter in a manner indicated by the contours of the $\mathcal{E}$ field.

surge back and forth producing a reading on the detector. If the receiving antenna is turned through 90 degrees around the direction of propagation, the detector reading drops to 0. In principle, two transmitters aligned perpendicular to each other, one with the current out of phase by 90 degrees with the other, transmit an electric field that is circularly polarized.

Common sources of visible and infrared light differ from radio and microwave sources in that the elementary radiators are the atoms and the molecules of matter which tend to act independently of one another. The light that propagates in a given direction consists of many independent wave–trains that vibrate in a random fashion relative to each other. The electric fields are thus randomly distributed in the plane normal to the direction of propagation; consequently the electric vector forms no well–defined pattern as the field oscillates. Despite this irregularity, we are still able to describe this random case in terms of two waves vibrating at right angles to each other. In this case the wave may be viewed as two linearly polarized waves with a random phase difference. Thus the fluctuations of one of these superimposed fields is independent of the other, and if the average amplitudes of the two are equal, then the light is said to be *unpolarized*. Sunlight, radiation emitted from the Earth, and light radiated from incandescent sources are examples of unpolarized radiation sources.

Radiation is said to be *partially polarized* when its state of polarization falls between the extremes of total polarization and complete unpolarization. Such radiation can be thought to be composed of varying contributions of two beams, one completely polarized and the other unpolarized. As we shall see, the degree to which a partially polarized beam is polarized can then be assessed in terms of the ratio of the intensity of this polarized component to the total intensity. This ratio is referred to as the *degree of polarization* and varies from 0 for completely unpolarized radiation to unity for a fully polarized beam.

## 2.4 Stokes' Parameters

The state of polarization is completely specified by the four parameters described earlier (two amplitude quantities, the magnitude, and the sign of the phase difference). While it is not difficult to devise ways of measuring amplitude quantities (since these are just related to intensities), it is an entirely different matter to measure phase differences. We now turn to an alternate description of the polarization of light which is tied directly to observable intensity quantities. These intensity quantities are the four *Stokes parameters*, $I, Q, U, V$, named after G. N. Stokes, who systematically studied polarized light fields in 1852. It is quite remarkable that a description of the myriad forms of polarized light reduce to the specification of just four parameters. Methods to measure these four parameters are described, and some attempt is made to connect these parameters to the more heuristic description of polarization given earlier.

Consider the simple, hypothetical instrument shown in Fig. 2.12. Light is collected by the instrument and passed through two special plates, a spectral filter, and ultimately down to the detector. The relative placement of the wave plate $W$ and the polarizer $P$ (used here as an analyzer) is important, although where the filter is relative to $W$ and $P$ is immaterial provided the properties of $W$ and $P$ are spectrally flat. We will suppose for the purpose of this discussion that $W$ and $P$ are ideal and that there is no attenuation of radiation when transmitted through either plate. In order to understand the workings of this instrument it is appropriate first to review some special properties of the materials used to construct $W$ and $P$.



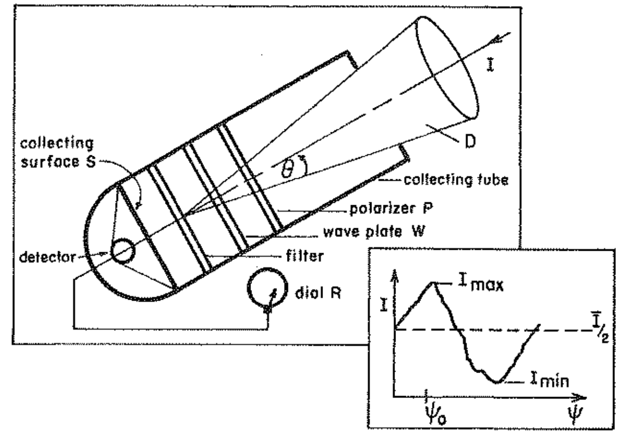


**Figure 2.12** Schematic details of a radiometer fitted with a polarizer $P$ and wave plate $W$ which is used to measure polarized radiation and to obtain the four Stokes parameters. A typical output of the instrument is shown as the polarizer is rotated through $\psi$.

When a transverse wave propagates through certain crytalline solids, the oscillations excited in the material[1] depend on the direction of polarization of the wave as well as its direction of propagation. The molecules of these solids tend to be oriented and unable to rotate about their equilibrium positions within the crystal lattice. For certain types of material, the ability of molecules to oscillate is not necessarily the same in all directions. This material is then said to be *anisotropic*. When an electromagnetic wave penetrates a slab of anisotropic material, no matter what the initial state of polarization, the electromagnetic wave behaves as two waves that are polarized at right angles to each other and propagate at different phase velocities. The material is then said to be *birefringent*.[2] Since the two

[1] The basic nature of these oscillations is the subject of much of Chapters 4 and 5.

[2] Many substances that are normally isotropic become anistropic and birefringent when subjected to mechanical stresses. This fact is useful in engineering design studies in that strains in gears, in bridge structures, and so on, can be studied quantitatively by stressing plastic models and examining the optical anisotropy that results.

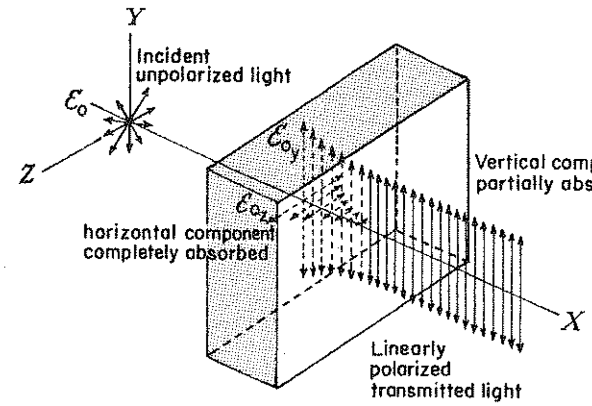


**Figure 2.13** An illustration of dichroism.

waves travel through the slab of crystal at different speeds, there will be a phase shift between them as they emerge from the crystal. If the slab thickness is chosen so that this phase difference is precisely 90 degrees (the retardation angle), then the slab is called a *quarter–wave plate*, and, according to our discussion in relation to Fig. 2.10, linearly polarized light will become circularly polarized as it emerges from the crystal. It is this type of crystal, in the form of a quarter–wave plate, that we use in our simple instrument for $W$.

Certain anisotropic substances may also absorb more in one direction than in another. An electromagnetic wave that propagates through a sufficiently thick piece of this material becomes gradually polarized in only one direction. This situation is called *dichroism* and is shown in Fig. 2.13. Dichroic materials offer a simple and inexpensive way of producing and analyzing polarized light. We use this material for the polarizing plate $P$.

The *optical axis* is a characteristic direction in the crystal that allows us to establish the orientation of the polarization of light on transmission through the crystal. The orientation of the optical axis of the polarizer in the instrument is important in the experiments described next.



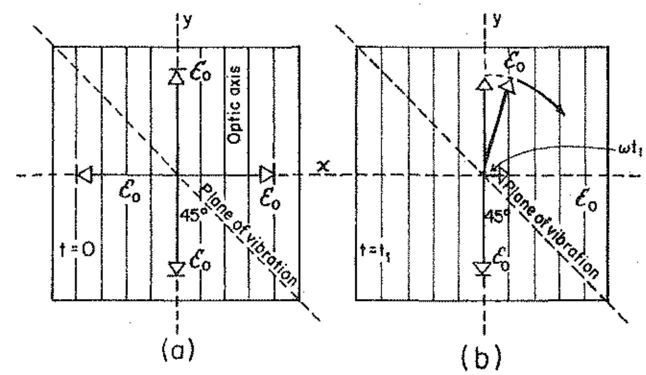


**Figure 2.14** (a) Plane polarized light falls from behind on a quarter–wave plate oriented so that light emerging from the page is circularly polarized. (b) In this case, the electric vector $\mathcal{E}$ rotates clockwise as seen by an observer facing the light.

## *Excursus: A Circular Basis for the Description of Polarization*

There is an alternate mathematical way to describe polarization other than the linear basis described earlier. Here we discuss the circular basis for polarization. Let us first contemplate our experiment using the radiometer of Fig. 2.12 where $W$ is a quarter–wave plate. Consider light entering this instrument and suppose that the optical axis of our wave plate is oriented at 45 degrees to the direction of polarization of the beam that emerges from the polarizer (see Fig. 2.14). The wave that emerges from the wave plate is circularly polarized and we will determine in what direction the electric vector rotates assuming the direction of propagation is out of the page for the geometric arrangement shown.

The component of wave vibrations parallel to the optical axis of the wave plate (we denote this direction as the $\ell$ axis) as the wave emerges from $W$ is

$$\mathcal{E}_\ell = \mathcal{E}_o \sin 45^\circ e^{i\omega t} = \frac{1}{\sqrt{2}}\mathcal{E}_o \cos(\omega t) \tag{2.25a}$$

where we assume for convenience that $x = 0, \phi_o = 0$ in (2.8). The wave component perpendicular to the optical axis is

$$\mathcal{E}_r = \mathcal{E}_o \cos 45^\circ e^{i\omega t - \pi/2} = \frac{1}{\sqrt{2}}\mathcal{E}_o \sin(\omega t) \tag{2.25b}$$

where the $-\pi/2$ phase shift represents the action of the quarter–wave plate. To decide the direction of rotation, we locate the tip of the rotating electric vector at two instants of time, say $t = 0$ and, at a short time later, $t = t_1$. At $t = 0$,

$$\mathcal{E}_\ell = \frac{1}{\sqrt{2}}\mathcal{E}_o \text{ and } \mathcal{E}_r = 0.$$

At $t = t_1$, these coordinates become approximately

$$\mathcal{E}_\ell = \frac{1}{\sqrt{2}}\mathcal{E}_o \cos\omega t_1 \approx \frac{1}{\sqrt{2}}\mathcal{E}_o(1 - \omega t_1)$$

$$\mathcal{E}_r = \frac{1}{\sqrt{2}}\mathcal{E}_o \sin\omega t_1 \approx \frac{1}{\sqrt{2}}\mathcal{E}_o(\omega t_1)$$

Thus the vector representing the emerging circularly polarized light is rotating clockwise when the observer faces the light source and the emerging light is therefore *left–circularly polarized.*

One important result of this analysis is that it demonstrates how a circularly polarized can be written as a combination of linearly polarized waves. For example, we write for left–hand circular polarization,

$$\mathcal{E}_{LH} = \frac{1}{\sqrt{2}}(\mathcal{E}_\ell - i\mathcal{E}_r) \tag{2.26a}$$

and similarly

$$\mathcal{E}_{RH} = \frac{1}{\sqrt{2}}(\mathcal{E}_\ell + i\mathcal{E}_r) \tag{2.26b}$$

for right–hand circular polarization. It is also relevant to note how a linearly polarized wave can be represented as the sum of right– and left–hand polarized waves (e.g., $\mathcal{E}_\ell = [\mathcal{E}_{RH} + \mathcal{E}_{LH}]/\sqrt{2}$). We can write the relationship between the circular and linear polarization basis as

$$\begin{pmatrix} \mathcal{E}_{RH} \\ \mathcal{E}_{LH} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}\begin{pmatrix} \mathcal{E}_\ell \\ \mathcal{E}_r \end{pmatrix}. \tag{2.27}$$



### 2.4.1 Measurement of I,Q,U,V

We will now perform two experiments with our radiometer. In the first experiment, the instrument operates without the wave plate, and the polarizer is initially aligned with the optical axis pointing along a specified reference direction (say along the vertical direction). The angle of the polarizer's optical axis measured from this reference direction is denoted by $\psi$. This angle is then systematically varied from zero radians along the reference direction in a clockwise direction (when looking along the direction of the traveling beam and into the instrument) to $\pi$ radians. A typical example of the intensities recorded by this instrument as $P$ rotates in this way is also shown in Fig. 2.12. If $I(\psi)$ is the intensity measured by the instrument for the given angle $\psi$, then we obtain

$$I(\psi) = \frac{1}{2}[\bar{I} + \Delta I \cos 2(\psi - \psi_o)] \tag{2.28}$$

where it simply follows that

$$\bar{I} = I_{max} + I_{min} \tag{2.29a}$$

$$\Delta I = I_{max} - I_{min} \tag{2.29b}$$

where $I_{max}$ and $I_{min}$ are the maximum and minimum readings, respectively, and $\psi_o$ is the angle corresponding to $I_{max}$. Introducing

$$Q = \Delta I \cos 2\psi_o \tag{2.30a}$$

$$U = \Delta I \sin 2\psi_o \tag{2.30b}$$

causes equation (2.28) to become

$$I(\psi) = \frac{1}{2}[\bar{I} + Q\cos 2\psi + U \sin 2\psi] \tag{2.31}$$

The second experiment repeats the procedure of the first experiment except now a wave plate of a fixed retardation $\epsilon$ is used. In this case, the intensity is

$$I(\psi, \epsilon) = \frac{1}{2}[\bar{I} + Q\cos 2\psi + (U\cos\epsilon - V\sin\epsilon)\sin 2\psi] \tag{2.32}$$

where $I(\psi, \epsilon)$ is the intensity measured for the polarizer angle $\psi$ and the wave plate retardation $\epsilon$.

Only four direct readings from our instrument are needed to obtain the Stokes parameters, $I, Q, U$, and $V$. These four intensity measurements are $I(0,0), I(\pi/2,0), I(\pi/4,0)$, and $I(\pi/4,\pi/2)$; the first three are obtained with the instrument configuration used in our first experiment, and the fourth measurement is carried out with the configuration of our second experiment with a quarter–wave plate for $W$.

The connection between the four intensity measurements and the four Stokes parameters is easily established by means of (2.32). Substituting the appropriate values of $\phi$ and $\epsilon$ in (2.32) produces

$$\begin{aligned} I(0,0) &= \frac{1}{2}[I+Q] \\ I(\pi/2,0) &= \frac{1}{2}[I-Q] \\ I(\pi/4,0) &= \frac{1}{2}[I+U] \\ I(\pi/4,\pi/2) &= \frac{1}{2}[I-V]. \end{aligned} \tag{2.33}$$

The relationship between the four Stokes parameters and the four intensity quantities can be written in the following mathematical way:

$$\mathbf{I} = \mathbf{P}\mathbf{I}_{obs} \tag{2.34}$$

where

$$\mathbf{P} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ -1 & -1 & 2 & 0 \\ 1 & 1 & 0 & -2 \end{pmatrix} \tag{2.35}$$

and where the (observable) intensity vector and the Stokes vector are

$$\mathbf{I}_{obs} = \begin{pmatrix} I(0,0) \\ I(\pi/2,0) \\ I(\pi/4,0) \\ I(\pi/4,\pi/2) \end{pmatrix} \quad \text{and} \quad \mathbf{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} \tag{2.36}$$

Thus $\mathbf{I}_{obs}$ and $\mathbf{I}$ are equivalent descriptions of polarized light fields in the sense that one allows the deduction of the other. There is



a difference, however, that is important. Our hueristic discussions of polarization tend to focus on the Stokes vector whereas $\mathbf{I}_{obs}$ is more suited to discussion of actual measurements and measurement systems. For these discussions, we adopt the short–hand notation $I_\ell$ for $I(0,0)$ to represent the intensity of the component parallel to the reference axis of the linear polarizer. In a similar way, we use $I_r$ for $I(\pi/2,0)$ for the intensity perpendicular to the reference direction. Further discussion of these components is given in Section 5.8.

Table 2.1 provides examples of $\mathbf{I}_{obs}$ and $\mathbf{I}$ for commonly occurring polarization states. We note from studying this table how the parameter $V$ specifies the extent of circular polarization with the sign of $V$ determining the handedness of this state. By contrast, $Q$ and $U$ characterize the extent to which the light field is linearly polarized; $Q$ refers to parallel and perpendicular polarization, and $U$ refers to polarization along the $\pm 45$ degree direction. If the light is unpolarized, then $Q = U = V = 0$, which naturally leads to a definition of the *degree of polarization* as $\sqrt{Q^2+U^2+V^2}/I$. Furthermore, the ratios $\sqrt{Q^2+U^2}/I$ and $V/I$ are referred to as the degrees of linear and circular polarization, respectively.

Only three of the Stokes parameters are actually independent for a single electromagnetic wave and are related by the identity

$$I^2 = Q^2 + U^2 + V^2 \tag{2.37}$$

In general, however, the light field that flows into our instrument is not a single continuous monochromatic wave; rather, it consists of many "simple waves" that flow in rapid succession, and the measurable intensities refer to a superposition of many millions of these simple waves with independent phases. Because the definition of Stokes parameters involves only intensities, it follows that the Stokes parameters of such a collection of incoherent beams (i.e., beams exhibiting no fixed relationship among the phases) are simply additive. Thus we understand that by Stokes parameters we mean the sums

$$I = \sum I_i, Q = \sum Q_i, U = \sum U_i, V = \sum V_i \tag{2.38}$$

where the index $i$ denotes each independent wave. It may be shown (but omitted for brevity) that under these circumstances

$$I^2 \geq Q^2 + U^2 + V^2 \tag{2.39}$$

Table 2.1 Radiance and Stokes Vectors of Common States.

| Verbal Description | Observable Intensity Vector | Stokes Vector [I,Q,U,V] |
|---|---|---|
| Vertically or parallel polarized intensity | $[1,0,\frac{1}{2},\frac{1}{2}]$ | [1,1,0,0] |
| Horizontally or perpendicular polarized intensity | $[0,1,\frac{1}{2},\frac{1}{2}]$ | [1,-1,0,0] |
| Linearly polarized intensity at +45° | $[\frac{1}{2},\frac{1}{2},1,\frac{1}{2}]$ | [1,0,1,0] |
| Linearly polarized intensity at -45° | $[\frac{1}{2},\frac{1}{2},0,\frac{1}{2}]$ | [1,0,-1,0] |
| Right circularly polarized intensity | $[\frac{1}{2},\frac{1}{2},\frac{1}{2},0]$ | [1,0,0,1] |
| Left circularly polarized intensity | $[\frac{1}{2},\frac{1}{2},\frac{1}{2},1]$ | [1,0,0,-1] |
| Unpolarized intensity | $[\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}]$ | [1,0,0,0] |

where the equality holds for completely polarized light and the inequality holds for partially polarized light.

## *Excursus: Example Calculation of I,Q,U,V*

We will now analyze our method for measuring the four Stokes parameters by presenting a calculation of the polarization state illustrated in Fig. 2.9e. As in problem 2.2b, we express the orthogonal components of the electric field as

$$\mathcal{E}_r = \mathcal{E}_o \cos(kx - ct)$$

$$\mathcal{E}_\ell = \mathcal{E}_o \cos(kx - ct + \frac{\pi}{4})$$

We choose our origin point as $x = 0$ and $t = 0$ for convenience. Let us write these components as

$$\mathcal{E}_r = e^{i0}$$

$$\mathcal{E}_\ell = e^{i\pi/4}$$



where we also assume $\mathcal{E}_o = 1$. The first intensity measurement $I(0,0)$ is

$$I(0,0) = \mid \mathcal{E}_\ell \mid^2 = \mid \frac{1}{\sqrt{2}}(1+i) \mid^2 = 1$$

and the second intensity measurement is

$$I(\pi/2,0) = \mid \mathcal{E}_r \mid^2 = 1$$

It follows from (2.35) that $I = I(0,0) + I(\pi/2,0) = 2$ and $Q = I(0,0) - I(\pi/2,0) = 0$. We derive the $U$ component from the third measurement with the polarizer aligned at 45 degrees with respect to the parallel direction. Thus the projection of the two orthogonal components onto this direction yields the amplitude

$$\mathcal{E}_{\pi/4} = \sin\frac{\pi}{4}e^{i0} + \cos\frac{\pi}{4}e^{i\pi/4} = \frac{1}{\sqrt{2}}(1 + e^{i\pi/4})$$

Therefore

$$I(\pi/4,0) = \frac{1}{2}(2+\sqrt{2})$$

and

$$U = 2I(\pi/4,0) - I = \sqrt{2}$$

The fourth intensity measurement is obtained with the instrument configured as shown in Fig. 2.12. The action of the wave plate in this configuration is to retard the $\mathcal{E}_r$ component relative to $\mathcal{E}_\ell$. Thus on passing through the wave plate, the orthogonal components are

$$\mathcal{E}_r = e^{-i\pi/2}$$

$$\mathcal{E}_\ell = e^{i\pi/4}$$

The projections of these at 45 degrees are

$$\mathcal{E}_{\pi/4} = \sin\frac{\pi}{4}e^{-i\pi/2} + \cos\frac{\pi}{4}e^{i\pi/4}$$

Therefore

$$I(\pi/4,\pi/2) = \frac{1}{2}(2-\sqrt{2})$$

and

$$V = -2I(\pi/4,\pi/2) + I = \sqrt{2}$$

From these calculations we obtain $\mathbf{I}= [2, 0, \sqrt{2}, \sqrt{2}]$ for elliptically polarized light. We also establish that the degree of polarization

$$\sqrt{Q^2 + U^2 + V^2}/I = 1$$

as expected.

## *Excursus: Mueller Matrices*

A beam of arbitrarily polarized radiation is represented by the column vector **I**. As this beam encounters an optical element (such as the polarizer or wave plate) the state of polarization changes. These changes are represented in terms of *Mueller matrices.* These matrices provide a convenient way of relating the incident and transmitted Stokes vectors. The usefulness of the Mueller matrices appears when assessing the effects of a series of optical elements on an incident beam. In this case, the combined effect of all elements is merely obtained from the product of their associated Mueller matrices. We will now consider the Mueller matrices of the elements of our radiometer since these matrices are also relevant to later discussions. The Mueller matrix for our ideal linear polarizer (e.g., Bohren and Huffman 1985) is

$$\frac{1}{2}\begin{pmatrix} 1 & \cos 2\psi & \sin 2\psi & 0 \\ \cos 2\psi & \cos^2 2\psi & \cos 2\psi \sin 2\psi & 0 \\ \sin 2\psi & \cos 2\psi \sin 2\psi & \sin^2 2\psi & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

where $\psi$ is the angle of the polarizer relative to the given reference direction. The Mueller matrix of our ideal polarizer for parallel transmission (i.e., for $\psi = 0$) is

$$\mathbf{M}_\ell = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.40}$$

The Mueller matrix of the wave–plate $W$ (i.e., of an ideal linear retarder) is

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & C^2 + S^2 \cos\epsilon & SC(1 - \cos\epsilon) & -S\sin\epsilon \\ 0 & SC(1 - \cos\epsilon) & C^2 + S^2\cos\epsilon & C\sin\epsilon \\ 0 & S\sin\epsilon & -C\sin\epsilon & \cos\epsilon \end{pmatrix}$$



where $C = \cos 2\psi, S = \sin 2\psi$. The Mueller matrix of W configured to measure $I(\pi/4, \pi/2)$ thus follows as

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

with $\epsilon = \pi/2$ and $\psi = \pi/4$. The combined effect of the linear polarizer and wave-plate is

$$\frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \tag{2.41}$$

If light of arbitrary polarization, specified by the four parameters $I, Q, U, V$, flows through this particular configuration of elements ($P$ first and $W$ next), the transmitted light emerges as 100% right circularly polarized and of the form

$$\mathbf{I} = \frac{1}{2}\begin{pmatrix} I + Q \\ 0 \\ 0 \\ I + Q \end{pmatrix}$$

Since matrix multiplication is not commutative, we deduce that the order of the elements is important to the final outcome.

### 2.5 Creation of an Electromagnetic Wave

There are three stages in the life of a photon: creation, propagation, and destruction. Both the theory of Maxwell and the discussion so far focus only on the propagation of the electromagnetic wave through space. Moreover, Maxwell's theory is formulated so that it is independent of the properties of matter. The creation and destruction of a photon, by contrast, occurs as a direct result of its interaction with matter. The final section of this chapter focuses on how electromagnetic waves are created through a process referred to as *emission.* As we shall see throughout this book, the relationship between the emitted radiation and the constituents of the atmosphere that emit this radiation is exploited in many remote sensing methods. For instance, emission properties are used to derive atmospheric temperature, to derive concentrations of certain trace gases, and in the estimation of precipitation among other important atmospheric properties. Thus, it is important in the context of remote sensing to understand how and why objects emit radiation.

### 2.5.1 Equilibrium Radiation and Kirchhoff's Law

The generation of electromagnetic waves occurs as a general result of accelerating electric charges. In general, any object is composed of a vast number of molecules. Even without the aid of external excitation, these molecules oscillate over a continuous range of frequencies and therefore emit radiation of all frequencies. However, this radiation is not emitted equally at all frequencies rather it is distributed according to the *emission spectrum* which, as we shall see, depends strongly on the temperature of the object.

The nature of the emission spectrum and its relationship to the temperature of the body loomed as a major challenge to physicists late in the nineteenth century. In fact, the relationship could not be accounted for by using the principles of classical physics, and its description marked one of the major turning points in the history of science. The hypothetical concept of a blackbody, (i.e., a body whose surface absorbs all radiation incident upon it) emerged in attempting to formulate the description of the emission spectrum. It also follows that any two blackbodies at the same temperature emit precisely the same radiation and that a blackbody emits more radiation than any other type of object at the same temperature.

It is more appropriate to view blackbody radiation as *equilibrium radiation*. One can appreciate this by considering an isolated cavity with walls opaque to all radiation. The cavity walls constantly emit, absorb, and reflect radiation until a state of equilibrium is reached (i.e., until the temperature of the cavity walls no longer change with time). This equilibrium radiation fills the cavity uniformly and is just the same as the radiation emitted by a hypothetical blackbody at the same temperature of the cavity. To understand why this is so, imagine that a blackbody is placed in the cavity. This body absorbs all of the equilibrium radiation incident on its surface and, since the cavity is in a state of equilibrium (it cannot cool or warm), the radiation emitted by the object must be precisely equal to that absorbed by it. This radiation happens to be the radiation that fills the cavity. Therefore, under the conditions of



equilibrium, the ability of a body to radiate is closely related to its ability to absorb radiation. The mathematical formulation of this statement is known as ***Kirchhoff's Law***, which can be written as

$$E_\lambda = a_\lambda \mathcal{B}_\lambda(T) \qquad (2.42)$$

where $E_\lambda$ is the emitted radiation and $\mathcal{B}_\lambda(T)$ is the radiation (expressed in intensity units) of the hypothetical blackbody. The proportionality constant is the absorption coefficient which varies between 0 and 1. If $a_\lambda = 0$, then (2.42) states that a body neither absorbs radiation at the given wavelength nor emits radiation at the same wavelength. For $a_\lambda = 1$, the emitted radiation is the blackbody radiation. As we shall see in Chapter 3, the absorption coefficient contains information about the type of matter that emits radiation and is therefore an important parameter in emission–based sensing methods. The wavelength dependence of this coefficient varies dramatically according to the nature of the matter emitting the radiation and the portion of the electromagnetic spectrum under consideration.

It is through the statement of Kirchhoff's Law that the whole point of blackbody radiation becomes apparent. All blackbodies at some temperature behave identically and the radiation emitted by such bodies at a given $\lambda$ depends only on the temperature of the body. Thus the emission of radiation at some chosen wavelength is solely determined by the characteristics of the emitting matter (through $a_\lambda$) and temperature (through $\mathcal{B}_\lambda$).

### 2.5.2 Planck's Blackbody Function and Related Laws

The theoretical question regarding the form of the wavelength distribution of the intensity of this cavity radiation and how this radiation in turn depends on the temperature of the walls of the cavity occupied the attention of many of the world's leading physicists during the 1890s. It was Max Planck who provided us with the theoretical description of the blackbody radiation however in doing so he was forced to make an assumption that proved to be one of the most daring departures from the philosophies of physics to that time. The assumption is that each oscillator in the walls of the cavity possesses only one of a discrete set of energies rather than the more conventional view that energy assumes any value above or equal to 0. The discrete energy level is given as

$$E = nh\nu \qquad (2.43)$$

where $n$ is an integer, and is referred to as *the quantum number* that defines the permitted number of discrete units of energy of the oscillator. The fundamental unit of energy turned out to be proportional to the frequency of the oscillator $\nu$ where the proportionality constant $h$ is known as *Planck's constant*. It is these discrete packets, or *quanta*, of energy that are emitted by the oscillators in the cavity walls after the oscillator undergoes a transition from one quantized energy state to another. On the basis of these arguments, Planck was able to demonstrate that the relationship,

$$\mathcal{B}_\lambda(T) = \frac{2hc^2}{\lambda^5(e^{hc/k_B\lambda T} - 1)} \qquad (2.44)$$

adequately describes blackbody radiation where $k_B$ is Boltzmann's constant and $T$ is the absolute temperature of the cavity walls.

Graphic examples of the function $\mathcal{B}_\lambda$, known as *Planck's function*, are given in Fig. 2.15 for three different temperatures. The three examples given demonstrate an obvious relationship. For example, consider an ordinary electrical element on a stove. On the highest and thus hottest setting the element glows brightest with a reddish hue (Fig. 2.16a). When the electricity is turned off and the element is allowed to cool, the color of the element fades until its luminosity vanishes; however it still radiates, a fact evident when a hand is placed above the cooling element. This simple experiment helps illustrate that the hotter the object the shorter the wavelength of the maximum intensity. This observation can be mathematically stated by an expression known as *Wien's displacement law* which establishes a connection between the wavelength of maximum emission ($\lambda_{max}$) and the temperature of the radiator. This law is simply derived from

$$\frac{\partial \mathcal{B}_\lambda}{\partial \lambda} = 0$$

from which it follows that

$$T\lambda_{max} = 2898 \quad (\mu m.K) \qquad (2.45)$$

Wien's displacement law is indicated on Fig. 2.15 as the solid, diagonal line joining the maxima of the three Planck functions. For example, at 6000 K, the maximum emission is in the blue region of the visible spectrum ($\lambda_{max} = 0.48$ $\mu$m) and substantial amounts of radiation are emitted across the entire visible spectrum. This distribution is very similar to the wavelength distribution of radiation



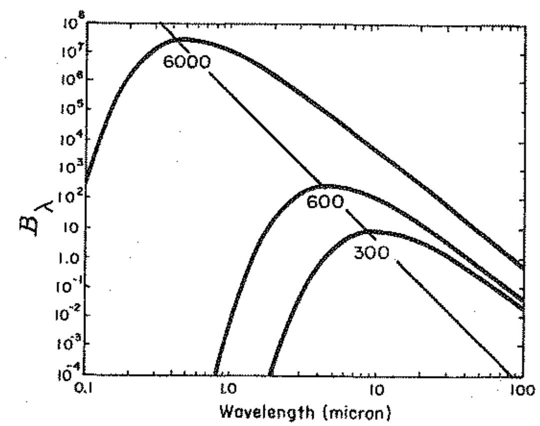


**Figure 2.15** Planck's blackbody curve at the three temperatures shown. The units of this function are given in $Wm^{-2}ster^{-1}$. The diagonal line intersecting the curves at their maxima depicts Wien's displacement law.

emitted by the sun. At temperatures more typical of the element of the stove, this maximum is shifted to the longer wavelengths so that more red light is emitted than blue light, whereas at 300 K, the maximum intensity occurs at far infrared wavelengths around 10 $\mu m$. Even at this temperature, however, radiation is still emitted at all wavelengths including visible light, although the emission of visible light is too dim to see with the naked eye (Fig. 2.16b).

Another obvious characteristic of blackbody radiation is the hotter the object, the greater the total amount of radiation emitted from a given surface area. This is just a statement of *Stefan–Boltzmann's law*, which derives from integration of $\mathcal{B}_\lambda$ over the entire wavelength domain,

$$B(T) = \int_0^\infty \mathcal{B}_\lambda(T)d\lambda = \frac{\sigma}{\pi}T^4, \qquad (2.46)$$

where $\sigma = 5.67\times10^{-8}$ $Wm^{-2}K^{-4}$ is the *Stefan–Boltzmann constant.* The radiation emitted by a 6000 K blackbody, for instance, is 160000 times that emitted by a 300 K blackbody.

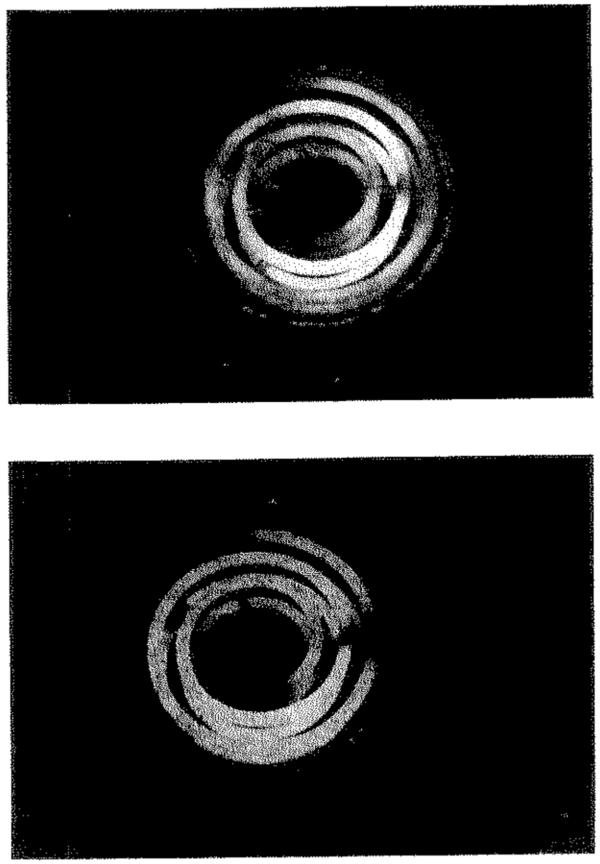

**Figure 2.16** The element of a stove with the heating control on the highest setting (upper panel). The bottom panel, like that for the upper panel, was taken in a darkened room but with the control turned down to the point where the element was no longer visible; the exposure time was 12 hours (from Bohren, 1987).



It is often convenient to use the Planck function defined in terms of wavenumber rather than wavelength. The relationship between different forms of the Planck function is obtained from the simple requirement that the energy integrated over the same spectral domain be equivalent. Thus,

$$\mathcal{B}_\lambda(T)d\lambda = -\mathcal{B}_{\tilde{\nu}}(T)d\tilde{\nu}$$

and, using (2.44) along with (2.2), it follows that

$$\mathcal{B}_{\tilde{\nu}}(T) = \frac{2hc^2\tilde{\nu}^3}{(e^{hc\tilde{\nu}/k_BT} - 1)} \tag{2.47}$$

There follows from either (2.44) or (2.47) two important limits of the Planck function. The first of these limits is *Wien's distribution* which applies as $\lambda \rightarrow 0$. In wavenumbers, this limit is

$$\mathcal{B}_{\tilde{\nu}} = 2hc^2\tilde{\nu}^3 e^{-hc\tilde{\nu}/k_BT} \tag{2.48}$$

The long wavelength limit $\lambda \rightarrow \infty$ is known as the *Rayleigh–Jeans distribution* and in terms of wavenumbers is expressed by

$$\mathcal{B}_{\tilde{\nu}} = 2k_BTc\tilde{\nu}^2 \qquad (2.49).$$

This longwave limit has a direct application to passive microwave remote sensing problems. At these wavelengths, the emission by the Earth's atmosphere is directly proportional to temperature. Thus, intensity is synonymous with temperature at these wavelengths.

From Table 2.2 we infer certain consequences of the long– and short–wavelength limits of the blackbody function and Wien's displacement law as they apply to atmospheric remote sensing. The relevant characteristics of the emission are tabulated at three wavelengths in the longwave spectrum: one in the shortwave limit at 4.3 $\mu$m, one at 15 $\mu$m, and the third in the longwave limit at 5 mm for the two temperatures indicated. The two shortest wavelengths lie in an atmospheric carbon dioxide absorption band and the third lies in a molecular oxygen absorption band. These wavelengths are suitable for temperature sounding (Chapter 7). The emission (in relative units) and the sensitivity of the emission to a small change in temperature (which can be thought of as related to the $d\mathcal{B}_\lambda/dT$) are listed. Based purely on the availability of energy, it seems that the 15$\mu$m region is superior given equivalent detector sensitivities

**Table 2.2** Properties of black body emission.

| Wavelength | 200 K | 300K |
|---|---|---|
| **Energy in relative radiance units** | | |
| 4.3 $\mu$m | 1.25 | 200 |
| 15 $\mu$m | 5000 | 15000 |
| 5 mm | 1 | 1 |
| **Temperature sensitivity*** | | |
| 4.3 $\mu$m | 1 | 20 |
| 15 $\mu$m | 10 | 6 |
| 5 mm | 4 | 1 |
| **Cloud transmission in %** | | |
| 4.3 $\mu$m | 6 | 1 |
| 15 $\mu$m | 1 | 1 |
| 5 mm | 96 | 99.98 |

* Relative to detector noise of 0.002 or 0.7 K.
From Smith (1972).

for each region. If a high temperature sensitivity is desirable, then the 4.3$\mu$m region is superior for detecting warmer temperatures but is still inferior to the 15$\mu$m region at colder temperatures, and the 5mm wavelength is the least sensitive of all three spectral regions. However, the optimum choice of wavelength is complicated by factors other than instrument sensitivity and available energy–the transmission characteristics of the atmosphere is one factor. This aspect is illustrated in the table in terms of typical transmission properties of clouds for the wavelengths under consideration (expressed as a percentage). We see that clouds in this example complicate matters further by strongly attenuating the shorter wavelengths, yet they are essentially transparent to millimeter wave radiation.

## *Excursus: Brightness Temperature*

We refer to the intensity expressed in units of temperature as the *brightness temperature*. This is the temperature that is required to match the measured intensity to the Planck blackbody function at the given wavenumber or wavelength. For microwave radiation this



is simply obtained from (2.49). At other wavelengths, the brightness temperature is obtained from either (2.44) or (2.47). For example, it follows from (2.47) that the brightness temperature $T_b$ is

$$T_b = \frac{hc\tilde{\nu}}{k_B}\left[\ln\left(\frac{2hc^2\tilde{\nu}^3}{I_m}+1\right)\right]^{-1} \tag{2.50}$$

where $I_m$ is the intensity measured at the characteristic wavenumber $\tilde{\nu}$ of the radiometer. The brightness temperature provides some indication of the temperature (and thus altitude in the atmosphere) at which the radiation is emitted. Some caution is needed here as the brightness temperature determined at one wavelength is generally different from that determined at another wavelength. Thus $T_b$ cannot be used to determine the total emission by the atmosphere. It is only the emission of a pure blackbody that is defined by a single temperature.

### 2.5.3 Blackbodies and Cavity Radiometers

The radiation emitted from cavities is a subject that is of more than just of heuristic interest as cavities are often an important element in the design of radiometers. In many instrument applications, it is important to provide a source of blackbody radiation. A particular configuration that closely approximates a blackbody is the conical cavity which directs any small amount of radiation reflected from the sides of the cavity further into the cavity (Fig. 2.17). If the internal surfaces of the cavity are coated with special highly emissive paint, then the absorption by the cavity is almost complete, and the radiation leaving the cavity is very nearly blackbody radiation emitted at the temperature of the cavity walls. This radiation can be accurately estimated from measurements of the cavity temperature.

Cavities of this type are used in two important but different ways. Cavities are used to provide a well–defined source of radiation for calibration of radiometers. In fact, many well–designed radiometers actually include a cavity as a means of internal calibration. In the second application, the highly efficient absorption properties of the cavity are exploited as a way of measuring the radiation that flows into the cavity. Cavity radiometers operating in this way have been flown on spacecraft to measure the radiation output of the sun.[3]

[3] This output is referred to as the solar constant which is defined as that radiation received on a horizontal surface at the top of the atmosphere when the Earth–sun distance is one astronomical unit.

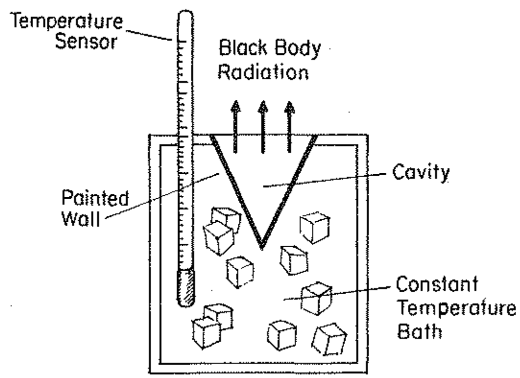


**Figure 2.17** A schematic illustration of the components of a simple calibration black body showing a conical cavity with a painted internal surface, a constant temperature sink which can be produced in a variety of different ways and a temperature measuring device.

An example of a cavity radiometer is shown in Fig. 2.18 which is a self–calibrating instrument aboard the ERBE satellite. The cavity design is slightly different from the one sketched in Fig. 2.17. It is composed of an inverted cone within a cylinder, the interior of which is coated with a specularly reflecting paint. The absorptivity of this particular cavity was determined to be 0.999 for the entire solar spectrum. Calibration is performed by turning the instrument to view space. The basic sensing of the instrument is the thermopile attached to the cavity, which effectively measures the temperature of the cavity after it absorbs the radiation. Other elements of the radiometer are the front baffle, which prevents any stray light from entering the instrument, and a constant temperature shield that alleviates any unwanted thermal influences on the cavity.

An important example of cavity radiometers on satellites are the radiometers flown on the Solar Maximum Mission (SMM), Nimbus 7, and ERBE satellites. Measurements obtained from these radiometers are used to study the variability of the solar irradiance. Measurements from SMM and Nimbus 7, collected over the past



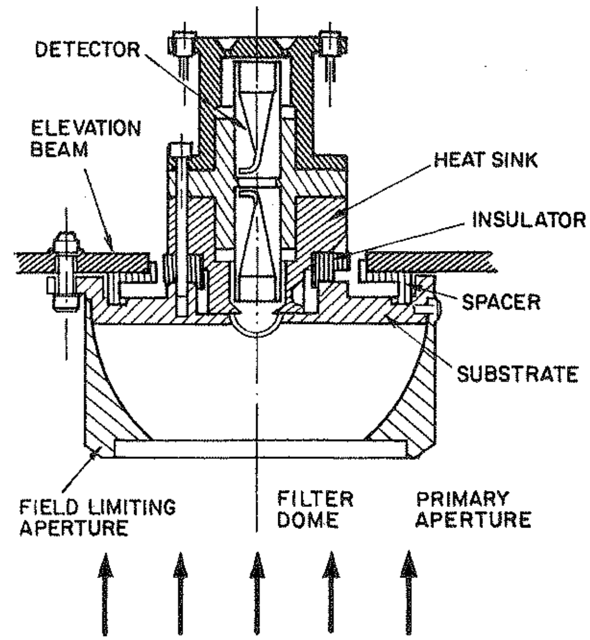


**Figure 2.18** Expanded view of the cavity radiometer flown on the ERBE satellite to measure solar output. Shown are the major components of the instrument described further in the text (Barkstrom and Smith, 1986).

10 years, show a long–term trend associated with the 11–year solar activity cycle with superimposed higher frequency variabilities (Fig. 2.19). The relationship between the sun's luminosity and the sunspot number is, at first glance, counterintuitive; the luminosity decreases with a decrease in the sunspot number. Solar luminosity is most affected by areas of bright faculae that are associated with sunspot activity and these bright areas outweigh the darker areas of the sunspots. Both sunspots and areas of faculae increase as solar activity increases.

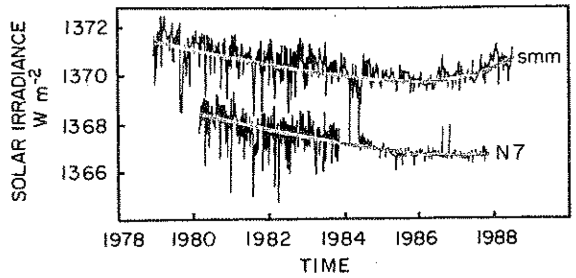


**Figure 2.19** The flickering of the sun is recorded by cavity radiometers flown on two satellites, Nimbus 7 and the SMM. On average (shaded line), the sun is brightest at time of maximum sunspot activity (adapted from Foukal, 1990).

## 2.6 Notes and Comments

2.1. Naming the regions of the spectrum, and the colors of visible light in particular, is somewhat arbitrary since there is really a continuum of "color". Newton distinguished seven colors in visible light: red, orange, yellow, green, blue, indigo, and violet.

2.2. Drain (1975) develops the Doppler shift given by (2.22) using a number of different arguments.

2.3. Polarization is described in elementary texts on electromagnetic radiation. A detailed account of polarization and its measurement is given by Kliger et al. (1990). They provide a brief historical account of polarization and discuss the pitfalls encountered in describing circular polarization. These arise from the definition of right versus left circular polarization and the definition of the sense of rotation relative to an observer. The convention used in this book is widely used in optics.

2.5. The history of the theory of black body radiation, culminating in Planck's famous radiation formula, is indeed fascinating. A brief account of this history can be found in Baggott (1992). In 1896 Wien developed a successful model of black body radiation at high frequencies, but his model failed at infrared frequencies. The model developed by Rayleigh in 1905, followed by a correction to that model by Jeans, led to the Rayleigh–Jeans law which was successful in matching observations at the lower frequencies (this is also sometimes referred to as the Rayleigh–Jeans catastrophe in the ul-



traviolet because of the model's failure at these higher frequencies). Planck's formula was able to fit the data at all frequencies although many physicists at that time were critical of the formula as merely providing an empirical fit to experimental data. Planck's concern at that time was to establish a theoretical basis for the formula which he was able to do through the use of thermodynamics.

Two blackbodies at the same temperature emit the same radiation. Proof of this lies in the second law of thermodynamics. In the case of two black surfaces A and B at the same temperature, suppose A radiates more energy than the other. Imagine placing these surfaces next to each other and allow each to absorb the radiation from the other. Thus B must absorb more radiation than it emits, receiving more energy and becoming hotter. A, correspondingly becomes cooler. Thus the second law of thermodynamics is violated and our assumption that A radiates more than B is false.

Why is the conical cavity a good blackbody? It is simply that light falling upon the hole enters the cavity and has little chance of emerging from it. Even if the absorption of the surface is not close to unity, almost all light is absorbed before any escapes the cavity.

In 1837, when Louis–Philippe was yet the King, the French physicist Claude Pouillet first measured and then named the solar constant. The following December, while the sun at Cape Town was near the zenith, Sir John Herschel tried his hand at the same measurement—with a crude apparatus—a thermometer encased in a small tin box filled with a measured amount of water, which, with the help of a black umbrella, was alternately shaded and then exposed to sunlight. The measured heating of the water provided a numerical definition of the most fundamental of climatic parameters: the precise amount of solar energy that falls upon the Earth.

Victorian interest in the solar constant evolved from the time of Herschel and Pouillet and focused not so much on its absolute value as on the possibility of predictable change, and the consequences of such changes for weather and climate. Specifically at issue in 1881 was whether the solar constant would increase, or decrease, with the coming and going of spots and other signs of activity on the solar surface. We have since been able to answer this question from the measurements collected on satellites (Foukal, 1990).

Before satellites, to answer this direct and simple question Balfour Stewart, a Scottish meteorologist, developed a new and more accurate actinometer, and in the 1880s sent it off to India, where, he reasoned, sunny skies would bring the nagging matter more quickly to an end. But little was achieved. After a few years, when British interest faded, the quest was taken up in America by Samuel Langley and later Charles Greeley Abbot, who for half a century doggedly pursued the answer, without success, from mountaintops around the world.

At the start of the present decade, after more than 140 years of effort, climatology still had no clear answer as to how the solar constant varied, or indeed whether Pouillet's choice of words to describe this quantity was right, after all. We now have a much better understanding of how this quantity varies with solar activity thanks largely to high quality satellite observations. An excellent overview of solar variability and a discussion of the satellite measurements of solar irradiance is given by Foukal (1990).

## 2.7 Problems

The following are physical constants useful for a number of problems given throughout this book: speed of light c=$2.99792\times10^8$ $ms^{-1}$, vacuum permittivity $\epsilon_o = 8.854 \times 10^{-12}$ $m^{-3}kg^{-1}s^2Co^2$ where Co is used for coulomb, the Stefan–Boltzmann constant $\sigma = 5.67051\times10^{-8}$ $Wm^{-2}K^{-4}$, Planck constant $h = 6.62608\times10^{-34}$ J s, Boltzmann's constant $k_B = 1.38066\times10^{-23}$ J $K^{-1}$.

2.1. Briefly explain or interpret the following:
   a. Could a sound wave in the air be circularly polarized?
   b. Unpolarized light falls on two polarizing sheets so oriented that no light is transmitted. If a third polarizing sheet is placed between them and arranged so that light is transmitted, can this transmitted light be polarized?
   c. Devise a way to identify the direction of the optical axis of a quarter wave plate.
   d. If we look into a cavity whose walls are maintained at a constant temperature no details of the interior are visible.
   e. Where and why in the atmosphere does Kirchhoff's law fail?
   f. The solar constant is not constant.
   g. The colors of stars are related to their temperatures whereas the colors of planets are not.

2.2. Describe the state of polarization represented by the following:
   a. $\mathcal{E}_x = \mathcal{E}_o \sin(kx - wt)$, $\mathcal{E}_y = \mathcal{E}_o \cos(kx - wt)$
   b. $\mathcal{E}_x = \mathcal{E}_o \cos(kx - wt)$, $\mathcal{E}_y = \mathcal{E}_o \cos(kx - wt + \frac{\pi}{4})$
   c. $\mathcal{E}_x = \mathcal{E}_o \sin(kx - wt)$, $\mathcal{E}_y = \mathcal{E}_o \sin(kx - wt)$



2.3. Electromagnetic radiation from the sun falls on the top of the Earth's atmosphere at the rate of $1.37\times10^3$ $Wm^{-2}$. Assuming this to be plane wave radiation, estimate the magnitude of the electric and magnetic field amplitudes of the wave. The units of the electric field are m kg $s^{-2}Co^{-1}$ and the units of the magnetic field are kg $s^{-1}Co^{-1}$ which is also known as a telsa (T).

2.4. Assume that a 100 W lamp of 80% efficiency radiates all its energy isotropically. Compute the amplitude of both the electric and magnetic fields 2 m from the lamp.

2.5. A plane, sinusoidal, linearly polarized electromagnetic wave of wavelength $\lambda = 5.0\times10^{-7}$m travels in a vacuum along the $x$ axis. The average flux of the wave per unit area is 0.1 $Wm^{-2}$ and the plane of vibration of the electric field is parallel to the $y$ axis. Write the equations describing the electric and magnetic fields of the wave.

2.6. Certain characteristic wavelengths in the light from a galaxy in the constellation Virgo are observed to be increased in wavelength, compared with terrestrial sources, by about 0.4%. What is the radial speed of this galaxy with respect to Earth? Is it approaching or receding?

2.7. The difference in wavelength between an incident microwave beam and one reflected from an approaching or receding automobile is used to determine its speed on the highway. (a) Show that if $v$ is the speed of the car and $\nu$ the frequency of the incident beam, the change in frequency is approximately $2v\nu/c$, where $c$ is the speed of the electromagnetic radiation. (b) For microwaves of frequency 2450 megacycles/sec what is the change of frequency per mile/hour of speed?

2.8. Calculate the Doppler shift (in Hertz) of radar and laser beams backscattered by particles with radial speeds of 0.01, 0.1, 1.0, and 10 $ms^{-1}$. Choose the wavelengths of 0.69 $\mu m$, 1.04 $\mu m$, and 10.6 $\mu m$ for the laser and 3 cm, 5 cm, and 10 cm for the radar. Compare these frequency shifts to the frequency of the carrier beam.

2.9. Determine the Stokes parameters for the states of polarization represented by (b), (d), and (e) in Fig. 2.10.

2.10. Determine the Stokes parameters of the $\mathcal{E}$ fields defined in Problem 2.2.

2.11. An infrared scanning radiometer aboard a meteorological satellite measures the outgoing radiation emitted from the Earth's surface at a wavelength of $10\mu m$. Assuming a transparent atmosphere, what is the temperature of the surface if the observed intensity is $0.98 \times 10^4$ ergs $cm^{-2}$ $\mu m^{-1}$ $sr^{-1}$?

2.12. What is the ratio of the spectral radiances of black bodies at 300 K and 6000 K at (a) 1 GHz, (b) 1000 GHz, (c) 1 $\mu$m, and (d) 0.1 $\mu$m?

2.13. Show that, for a black body, the wavelength at which $B_{\tilde{\nu}}$ is maximum is about 1.76 times greater than the wavelength at which $B_\lambda$ is maximum at the same temperature.

2.14. Derive the Rayleigh–Jeans distribution (2.49) from (2.47).

2.15. Find the wavelength at which the incoming solar irradiance at the top of the Earth's atmosphere is equal to the outgoing terrestrial irradiance. Assume the sun and Earth to be emitting as blackbodies at 6000 K and 255 K, respectively. The radius of the Earth and the Earth–sun distance are given in Prblem 6.3.

# 3

# Microscopic Interactions — Atomic and Molecular Absorption

Our ability to interpret measurements and subsequently infer information about the atmosphere from them could not be possible without some knowledge of the way that electromagnetic radiation interacts with matter and, in turn, not possible without some understanding of the fundamental properties of matter itself. It is certainly beyond the scope of this book to provide a thorough discussion of such a basic topic. The focus of this and the next two chapters concerns the properties of matter that are most relevant in its interaction with radiation. The specific topics discussed in this chapter apply to the microscopic scale (i.e., how radiation interacts with matter on the atomic and molecular level) and primarily concentrate on the absorption of radiation by gases in the Earth's atmosphere.

## 3.1 The Atomic Absorption Spectrum

### 3.1.1 The Bright Line Spectrum

The concept of a blackbody was introduced in Chapter 2. It was shown how the emission by a solid body produces a spectrum that is continuous in wavelength and, temperature aside, independent of the nature of matter. However, this is hardly the complete story. Almost two centuries before Planck's description of blackbody radiation, it was noted how vaporization of certain volatile metals produces distinct colors in flames and how these colors clearly relate to the type of metals being vaporized. In 1752 Thomas Melvill studied the color of the flame after its light was passed through a prism and discovered that the spectrum was not continuous like that of the sun or of the radiation emitted from cavities; rather, it existed as a series of distinct bright lines, the spectral locations of which varied according to the particular substance placed in the flame. This work, along with an instrument developed in the early part of the nineteenth century (the *spectroscope*) to observe the spectrum, heralded the modern science of *spectroscopy*. Analyzed in the spectroscope, the spectra of